RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2017-20]

B.A./B.Sc. FIRST SEMESTER (July – December) 2017

Mid-Semester Examination, September 2017

Date : 12/09/2017 **PHYSICS (Honours)**

Time : 11 am – 1 pm Paper : I Full Marks : 50

(*Answer any <u>five questions</u> taking at least <u>one from each group</u>*) [5×10]

<u>Group – A</u>

1. a) A fluid moves so that its velocity at any point is $\vec{V}(x,y,z)$. Show that the loss of fluid per unit volume per unit time in a small parallelepiped having centre at p(x,y,z) and edges parallel to the co-ordinate axes and having magnitude $\Delta x, \Delta y, \Delta z$ respectively is given approximately by $\vec{\nabla}.\vec{V}$. [4]

   b) Find $\vec{\nabla}\times\vec{r}\,f(r)$ where f(r) is differentiable. [2]

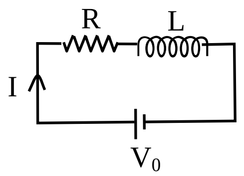
   c) Calculate the divergence and curl for the vector $\vec{A} = r^n\hat{r}$. [2+2]

2. a) Show that the greatest rate of change of ϕ takes place in the direction of and has the magnitude of the value $\vec{\nabla}\phi$. [2]

   b) Given, $\vec{A} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$. Is these a function ϕ such that $\vec{A} = -\vec{\nabla}\phi$? If so, find ϕ. [3]

   c) Find a general solution to the differential equation $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = \log x\,;\ x \in (0,1)$. [5]

3. a) From Kirchoff's law, the current I in a circuit containing resistance R and inductance L in series with a constant voltage source V_0 obeys the equation. $L\dfrac{dI(t)}{dt} + RI(t) = V_0$. Find I(t). Given initial condition I(0) = 0. [4]



   b) Solve the non-homogeneous ODE $\dfrac{d^2y}{dx^2} + y = \sec x$. [4]

   c) $y_1 = \sin\omega x,\ y_2 = \cos\omega x$

      i) Show that y_1 and y_2 are dependent.

      ii) Find the Wronskian of these two functions. Justify the answer of (i) w.r.t the Wronskian. [2]

<u>Group – B</u>

4. a) How does the unit vectors in plane polar coordinate $(\hat{r},\hat{\theta})$ differ from the unit vectors in Cartesian coordinates $(\hat{i},\hat{j})$. Obtain the expressions for velocity and acceleration of a particle in plane polar coordinates. [5]

   b) The path of motion of planets around the sun is elliptical with the sun at one of the foci. If a coordinate system is defined with the sun at the origin and the major axis as the polar axis, then the position coordinates r and θ of the planet are related through $r = a(1-\in^2)/(1-\in\cos\theta)$ where $\in$ is the eccentricity of the ellipse. Find expressions for velocity and acceleration of the planet. [5]

5. a) Derive the work-energy theorem for a particle of constant mass moving under a force filed $\vec{F}$. Show that if $\vec{F}$ is conservative, a potential energy function V can be defined such that the total mechanical energy is conserved. [3]

b) Let S' be an inertial frame moving with uniform velocity $\vec{v}$ relative to an inertial frame S. If in a two particle collision, the total linear momentum is conserved in S, show that it is also conserved in S'. [3]

c) A particle is projected vertically upwards with an initial speed u in a medium that offers resistance kv^2 per unit mass where v is the instantaneous speed. Set up the equation of motion and find the maximum height reached. [4]

Group – C

6. a) Consider a resistive force proportional to the velocity of a oscillating mass. Write its differential equation and solve it.

b) Define normal modes and normal frequency of a vibrating system.

c) Draw Lissajous figure for : $x = A\cos\omega t,\ y = A\cos 2\omega t$

7. a) Derive the differential wave equation for plane waves of constant waveform in one dimension. [3]

b) Let y = $\sin kx \cos \omega t$, prove whether it is a solution of wave or not. [2]

c) Define group velocity and phase velocity. Find a relation between them. When these two velocities give same velocity? [2+3]

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